Partial Derivatives with TI-Nspire™ CAS

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TI-Nspire CAS does not have a function to calculate partial derivatives. Nevertheless, recall that to calculate a partial derivative of a function with respect to a specified variable, just find the ordinary derivative of the function while treating the other variables as constants. For example, suppose we have the function $g(x, y) = 2x + 2y$. To find the partial derivative of g with respect to x, treat y as a constant and take the derivative of $g(x, y)$ with respect to x: $\frac{d}{dx}(g(x, y))$. Likewise, to find the partial derivative of g with respect to y, treat x as a constant and take the derivative of $g(x, y)$ with respect to y: $\frac{d}{dy}(g(x,y))$.

Thus, to calculate the partial derivative of a function of two or more variables, use the *derivative()* function or the derivative template: *derivative(f(x,y),x)* or $\frac{d}{dx}(f(x, y))$ calculates the first partial derivative of $f(x, y)$ with respect to x and *derivative(f(x,y),y)* or $\frac{d}{dy}(f(x, y))$ calculates the first partial derivative of $f(x, y)$ with respect to y.

Example

a. Define functions for and calculate the first partial derivatives of $f(x, y) = \sqrt{x^2 + y^2}$. Define the functions to facilitate calculating the second partial derivatives or to evaluate the partial derivatives at a given point (x, y).

   Define $f(x, y)$:

   $f(x,y):=\sqrt{x^2-y^2}$ Done

   Define a function for $\frac{\partial f}{\partial x} = f_x$ and display the definition:

   $f_x(x,y):=\frac{d}{dx}(f(x,y))$ Done

   $f_x(x,y)$ $\frac{x}{\sqrt{x^2-y^2}}$

   Define a function for $\frac{\partial f}{\partial y} = f_y$ and display the definition:

   $f_y(x,y):=\frac{d}{dy}(f(x,y))$ Done

   $f_y(x,y)$ $\frac{-y}{\sqrt{x^2-y^2}}$

b. Define functions for and calculate the four second partial derivatives of $f(x, y)$:

Define a function for $\frac{\partial f}{\partial xx} = f_{xx}$ and display the definition:

$$f_{xx}(x,y) := \frac{d}{dx}\left(f_x(x,y)\right)$$

Done

$$f_{xx}(x,y) \qquad \frac{-y^2}{\left(x^2-y^2\right)^{\frac{3}{2}}}$$

Define a function for $\frac{\partial f}{\partial xy} = f_{xy}$ and display the definition:

$$f_{xy}(x,y) := \frac{d}{dy}\left(f_x(x,y)\right)$$

Done

$$f_{xy}(x,y): \qquad \frac{x \cdot y}{\left(x^2-y^2\right)^{\frac{3}{2}}}$$

Define a function for $\frac{\partial f}{\partial yy} = f_{yy}$ and display the definition:

$$f_{yy}(x,y) := \frac{d}{dy}\left(f_y(x,y)\right)$$

Done

$$f_{yy}(x,y) \qquad \frac{-x^2}{\left(x^2-y^2\right)^{\frac{3}{2}}}$$

Define a function for $\frac{\partial f}{\partial yx} = f_{yx}$ and display the definition:

$$f_{yx}(x,y) := \frac{d}{dx}\left(f_y(x,y)\right)$$

Done

$$f_{yx}(x,y) \qquad \frac{x \cdot y}{\left(x^2-y^2\right)^{\frac{3}{2}}}$$

Note that for $f(x, y)$ above, the mixed partial derivatives, f_{xy} and f_{yx} are equal:

$$f_{xy}(x,y) = f_{yx}(x,y) \qquad \text{true}$$

This is the case when the mixed partial derivatives of **f(x$_0$,y$_0$)** exist and are continuous in a (possibly small) open disk around the point (Clairaut's Theorem).

Partial derivatives for functions of more than two variables are calculated in the same manner.

The graph of $f(x, y) = \sqrt{x^2 + y^2}$: